Green as Grass

It has been raining so much that my grass has grown to inches tall. That is way too high. I would like to cut it down to inches tall. I was wondering if I could just cut 5 inches off at once.



In my local hardware store the clerk told me that you should never cut more than of the length of the grass to prevent too much stress on the roots. Otherwise the grass may die. I was also told to wait three days between cuts.



I figured out that three cuts should be enough, but that cutting off of the length by the third cut would make my grass too short.



Questions:

For questions a) and b) assume that the grass does not grow measurably between cuts.

1. If I keep cutting by of the length of the grass every three days, then how much shorter is my grass than inches after three cuts?



1. If I want my grass to be exactly inches by the third cut, then what part of the length of the grass should I cut the third time?



For questions c) and d) assume that the grass does grow in between cuts as indicated.

1. Assume that during a certain time of the year the grass grows 10% of its length between cuts. After how many cuts will the grass be too short? What part of the grass should you cut on the cut before to get to  inches exactly?
2. During another part of the year the grass is growing between cuts at a rate such that after three cuts you are exactly at  inches. What was the rate of growth during this time?

Solution(s)

In this problem students must realize that they cannot simply remove  inches each time. The amount cut off depends on the length of the grass. Each time we cut off of the current length of the grass.



1. If I keep cutting by of the length of the grass every three days, then how much shorter is my grass than inches after three cuts?



|  |  |  |
| --- | --- | --- |
| Number of cuts (c) | Calculation (Initial length ”) | New grass length (L) |
| 1 | - ( x ) = - = 5”  alternatively:  x = 5” | 5” |
| 2 | 5 – (5 x ) = 5 - = ”  alternatively:  5 x = = ” | ” |
| 3 | - ( x ) = - = ”  alternatively:  x = = ” | ” |

Note that a more advanced student could multiply by , rather than subtract of the length. These (expert) students may also see that the following function describes this process:





The answer to question a) is determined by finding the difference between and . This problem can be simplified by finding the following difference: .



This difference is inches (almost of an inch short).



b) If I want my grass to be exactly inches by the third cut, then what part of the length of the grass should I cut the third time?



In part b) students must realize that we want to find the difference between inches and inches ( inches) as a factor of inches.



This can be solved from the following equation:



Solving for *r* yields:



So for the third cut we need to remove of the remaining length.



c) Assume that during a certain time of the year the grass grows 10% of its length between cuts. After how many cuts will the grass be too short? What part of the grass should you cut on the cut before to get to  inches exactly?

The growth formula from part a) needs to be adjusted for the grass growth between cuts. A growth of 10% results in a growth factor of 1.1. Note that there is one less growth period that the number of cuts. The formula becomes as follows:

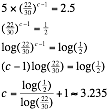


By making a table we can discover that by the third cut the height is approximately 2.69 inches and by the fourth cut approximately 1.97 inches. Thus we need to adjust our fourth cut.

Because the value of exponent c is not large we can use a table, but that is not an efficient method in many cases. We need a more general way.

Solve the following for *c*: 

This goes as follows:



From this calculation it is clear that I need to adjust my fourth cut as well.

To calculate what part of the grass I should cut on my fourth cut I need to first add 10% to the length of my third cut and then express the difference between this and 2.5 inches as a percentage of this length:



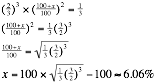
So I need to cut a little more than 1/7 of the length on the fourth cut.

d) During another part of the year the grass is growing between cuts at a rate such that after three cuts you are exactly at  inches. What was the rate of growth during this time?

We need to solve the following equation to find the growth rate such that by the third cut the remaining length is exactly 2.5 inches.



This goes as follows:



For all practical purposes we can state that the growth rate had to be 6% to be able to cut the grass to 2.5 inches after three cuts.